Order-sorted intensional logic

Expressing subtyping polymorphism with typing assertions and quantification over concepts

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Logic and subtyping polymorphism

- Consider the natural language sentence:

  An animal produces sound iff it produces a sound characteristic for its species.

- What we mean:

  An animal *produces sound* iff

  that animal is a dog and it barks,

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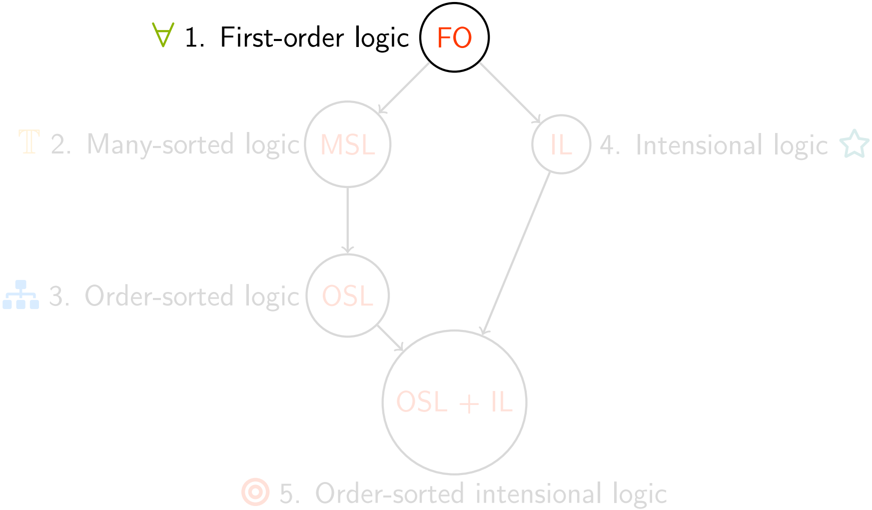
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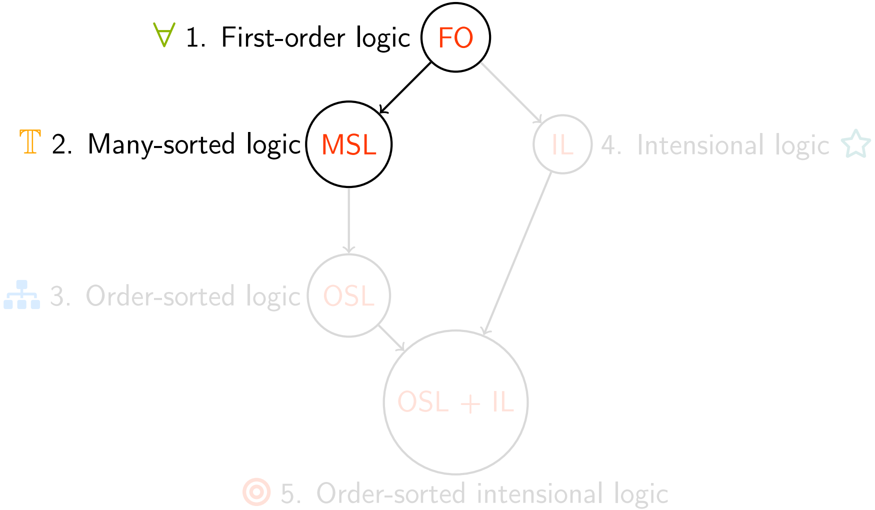
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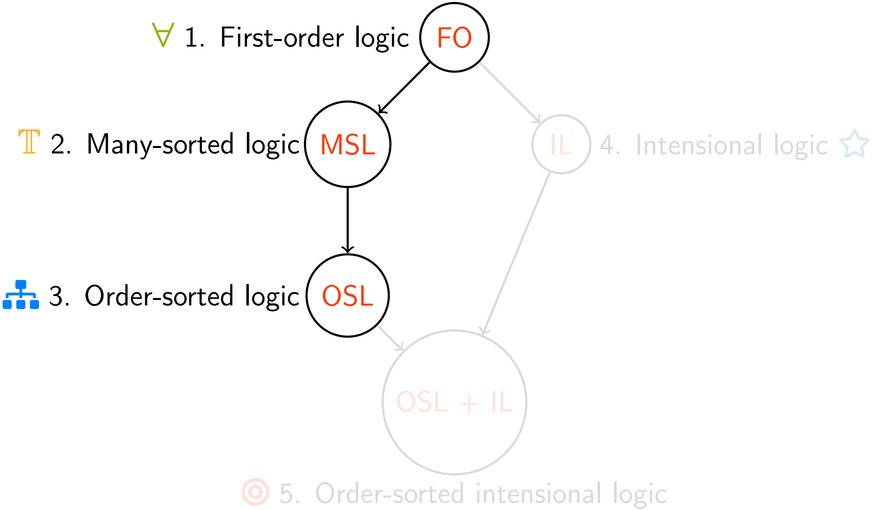
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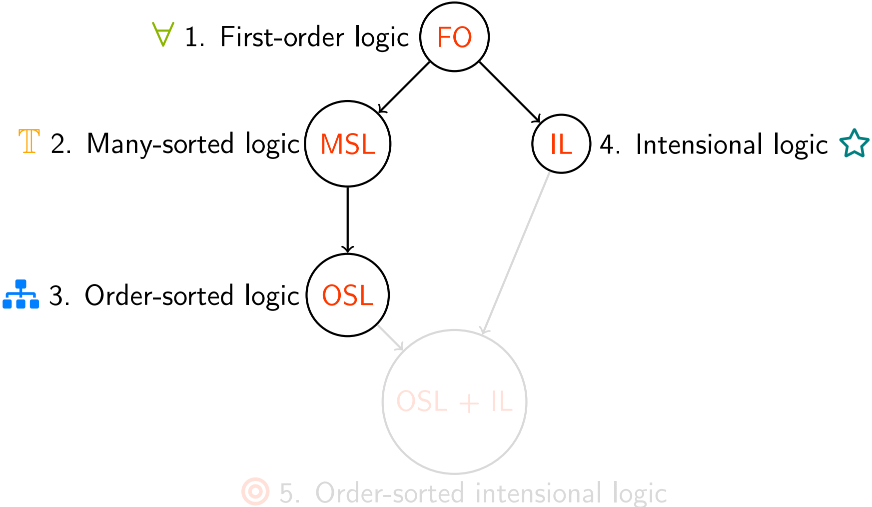
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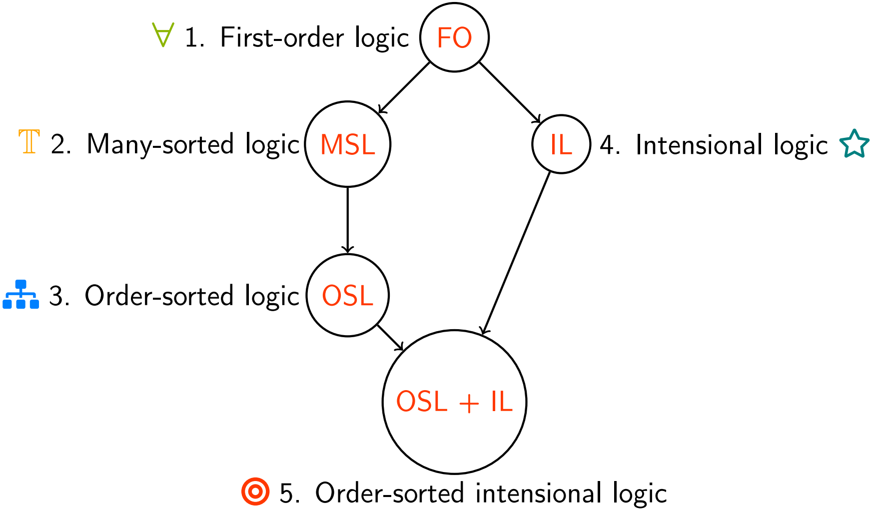
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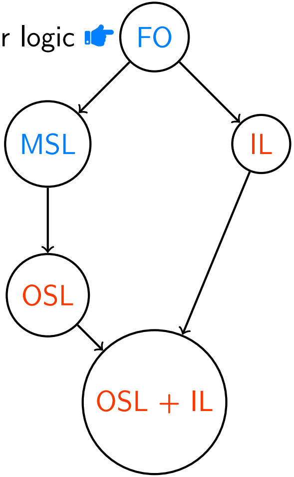
$\forall$ 1. First-order logic

$\mathbb{T}$ 2. Many-sorted logic



$\forall x \in \mathbb{T}$ First-order and many-sorted logic

- Example:

"There is a *Cat* eating."

- **First-order logic** is not typed! Each object is part of the "Universe".

$$\exists c : Cat(c) \land Eats(c).$$

- In **many-sorted logic**, we can introduce sort/type *Cat*.
- Then we can declare eats predicate as: $Eats : (\underline{Cat}) \to \mathbb{B}$
- Finally, we express the example statement as:

$$\exists c[Cat] : Eats(\underline{c}).$$

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- How to express that there is a *Dog* eating?

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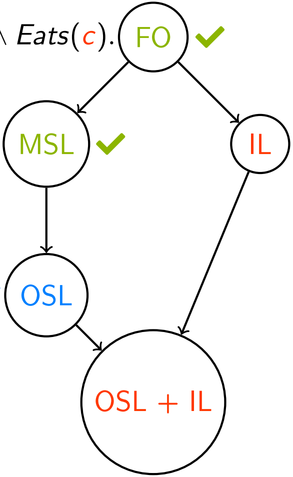

Order-sorted logic

- Hierarchy of types ($<:$ subtype relation):

$$\mathit{Dog} <: \mathit{Animal} \qquad \mathit{Cat} <: \mathit{Animal}$$

- Predicative form:

$$\mathit{Dog} : (\underline{\mathit{Animal}}) \to \mathbb{B}$$

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- Back to the eating example:

$$\mathit{Dog} <: \mathit{Animal} \qquad \mathit{Cat} <: \mathit{Animal}$$

$$\mathit{Eats} : (\underline{\text{Animal}}) \to \mathbb{B}$$

- "There is an *Animal* eating" is expressed as:

$$\exists a[\mathit{Animal}] : \mathit{Eats}(\underline{\text{a}}).$$

- Both of the following statements are well-typed:

$$\exists c[\mathit{Cat}] : \mathit{Eats}(\underline{\text{c}}). \qquad \exists d[\mathit{Dog}] : \mathit{Eats}(\underline{\text{d}}).$$

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- Any problems?
- Given predicates:

$$Bark : (\underline{Dog}) \to \mathbb{B} \qquad Meow : (\underline{Cat}) \to \mathbb{B}$$

- Can we define the predicate:

$$ProducingSound(\underline{Animal}) \to \mathbb{B}$$

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- Yes we can!

$$\forall a[Animal] : ProducingSound(\underline{a}) \Leftrightarrow (\exists d[Dog] : \underline{a} =^{*1} \underline{d} \wedge Bark(\underline{d})) \vee (\exists c[Cat] : \underline{a} =^{*2} \underline{c} \wedge Meow(\underline{c})).$$

- Can we do better?

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Guarded order-sorted logic

- Let: $S <: T$ $\quad S : (\underline{T}) \to \mathbb{B}$ $\quad a : T$
- Then: $S(a) \Rightarrow \phi[a]$ and $S(a) \land \phi[a]$ are well typed formulas!
- Abbreviated: $\langle\langle \phi(a) \rangle\rangle$ and $[[\phi(a)]]$
- Example:

$$\forall a[Animal] : ProducingSound(\underline{a}) \Leftrightarrow (Dog(\underline{a}) \land Bark(\underline{a})) \lor (Cat(\underline{a}) \land Meow(\underline{a})).$$

- The compact version:

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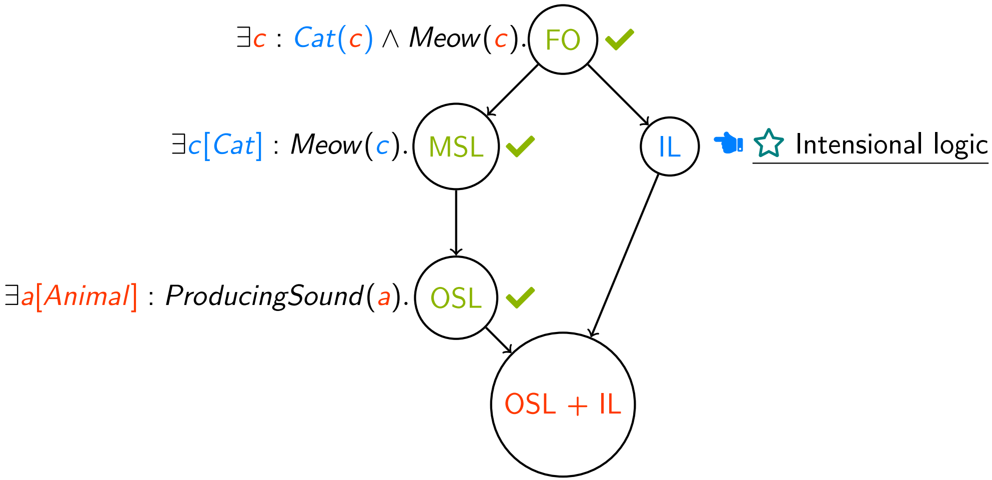
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Intensional logic

- Extension vs Intension
- Example: Evening star and morning star are concepts.
- New type: $Concept$
- Elements of $Concept := \{\widetilde{Animal}, \widetilde{Dog}, \widetilde{Cat}, \widetilde{Eats}, \widetilde{Bark}, \widetilde{Meow}, \ldots\}$
- New operators: $`(Animal) = \widetilde{Animal}$ and $\$(\widetilde{Eats})(d) = Eats(d)$
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$$Sound(Concept) \to \mathbb{B}$$

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- Back to the *ProducingSound*:

$$\forall a[Animal] : ProducingSound(\underline{a}) \Leftrightarrow (Dog(\underline{a}) \land Bark(\underline{a})) \lor (Cat(\underline{a}) \land Meow(\underline{a})).$$

- Can we do better with intentional logic?

$$\forall a[Animal] : ProducingSound(a) \Leftrightarrow \exists s[Concept] : Sound(s) \land \$(s)(a).$$

- This is not good!

$$\forall a[Animal] : ProducingSound(a) \Leftrightarrow Bark(\underline{a}) \lor Meow(\underline{a}).$$

- Can this be made correct in intensional logic? Yes!

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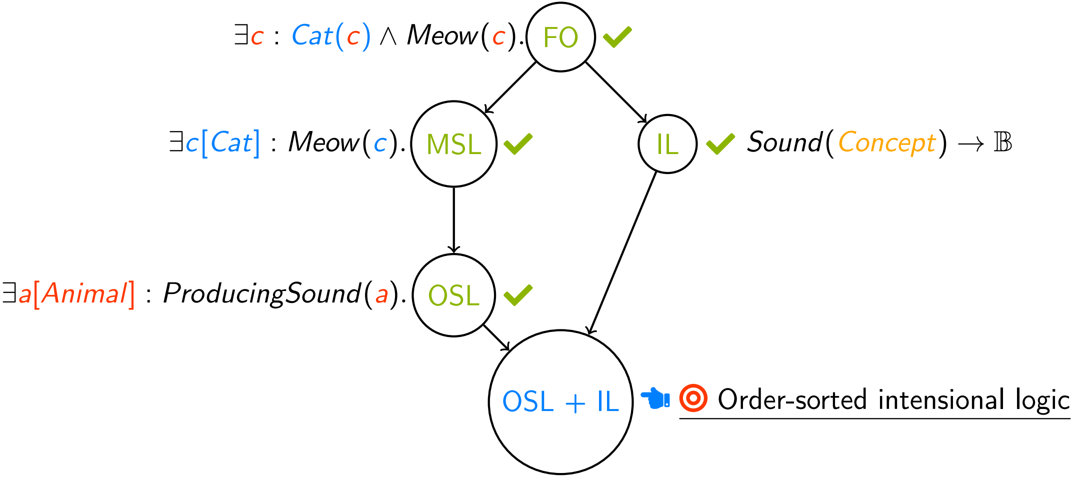
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- Introduce new type:

$$Sound <: Concept \quad := \{\widetilde{Bark}, \widetilde{Meow}\}$$

- Then *ProducingSound* can be defined as:

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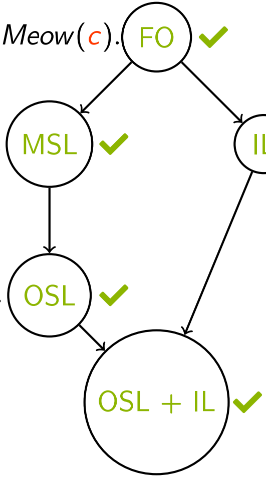
Conclusion



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$\exists c[Cat] : Meow(c).$

$Sound(Concept) \to \mathbb{B}$

$\exists a[Animal] : ProducingSound(a).$

$\forall s[Sound] : \exists a[Animal] : [\![\$(s)(a)]\!].$

Conclusion

- Expressing subtyping polymorphism is a challenge in order-sorted logic.
- Intensional logic is required.
- Implicit guarding makes it compact and elegant.
- Complex well-typing relation (future work).

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Takeaway: *Implicit type assertions and quantification over concepts provide elegant solution to subtyping polymorphism in OSL.*

Thank you for your attention! Questions?

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