An epistemic logic for modeling decisions in the context of incomplete knowledge

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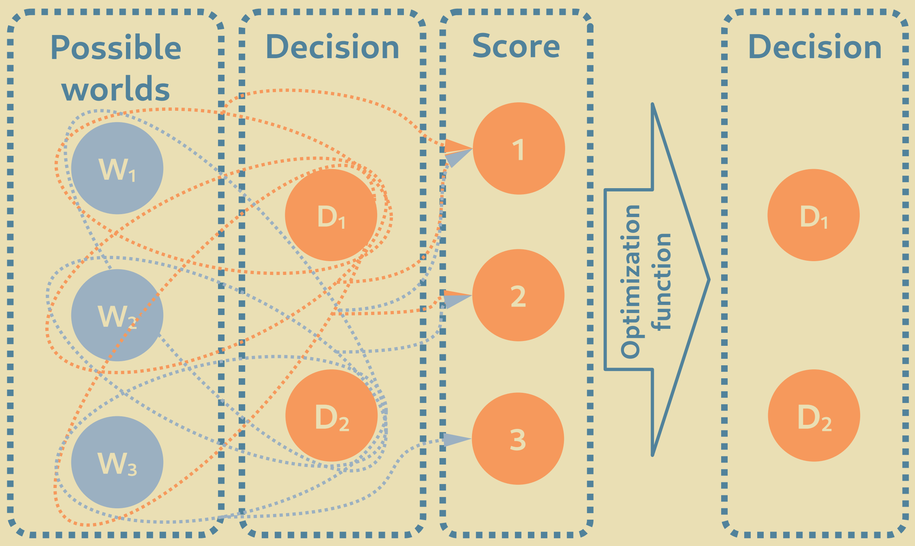
1 How do we model decision making processes?

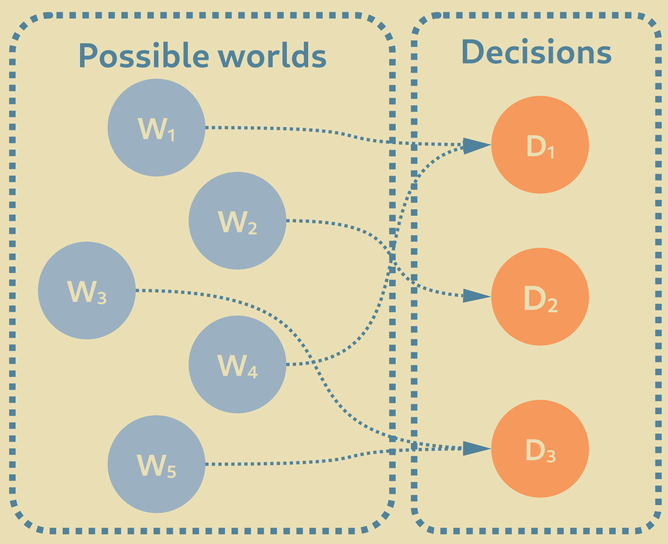
Decision theory [1]



Utility function - mapping possible worlds and decisions to an ordinal. Then the decision is made based on the **preference relation**.

⚠ Too complex (modeling-wise)

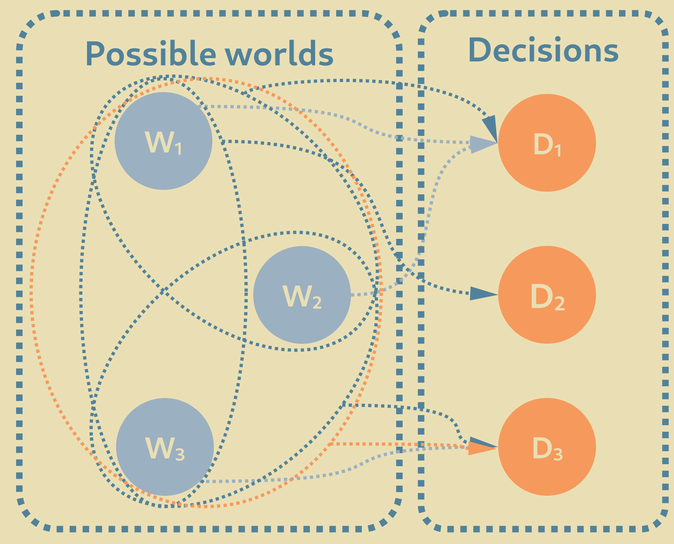
Rule-based systems



Generalization of notations like **DMN** - objective states of affairs are directly related to decisions.

⚠ No support for incomplete knowledge!

Epistemic decisions



Epistemic decisions – mapping all possible epistemic states (sets of objective state of affairs) to decisions.

✓ Relation between knowledge and decision

2 Decision Model and Notation [2]

Customer greeting example:

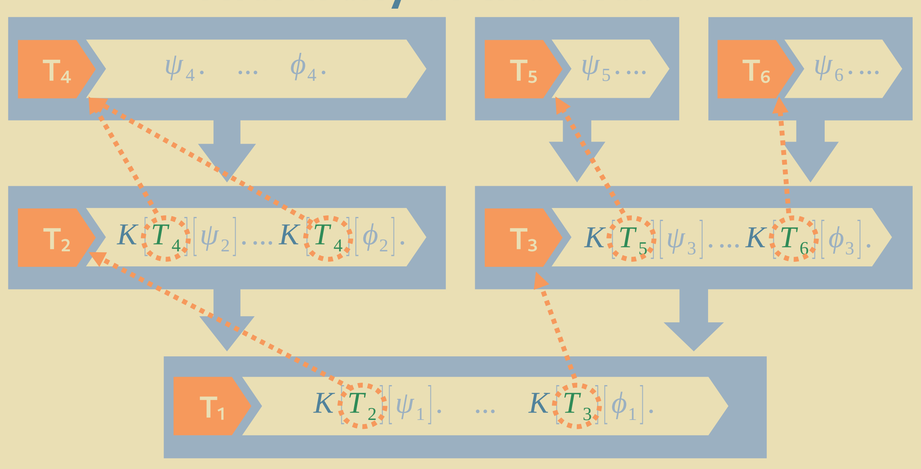
Salutation table			
U	Gender	Marital status	Salutation
1	Male	-	Mr
2	Female	Single	Ms
3	Female	Married	Mrs

It is impossible to make a decision when some data is unknown.

- "-" stands for the absence of the constraint on the value.
- In DMN it is impossible to make a decision if some value is unknown: For example, the marital status.
- Practically, it is possible to make decision if gender is known to be "male" since it does not depend on marital status.

3 Ordered Epistemic Logic [3]

Hierarchy of theories



- $T_1 \dots T_6$ - FO(ID) Theories, over the same vocabulary.
- ***K*** operator refers to other theories.
- K-**referencing** is a partial order.
- $T_4 \dots T_6$ – Do not contain the ***K*** operator, they represent knowledge of objective state of affairs.

4 Result - epistemic DMN

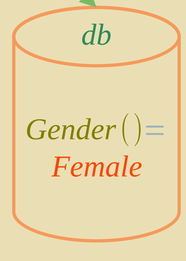
Greeting example in eDMN

Salutation table			
U	Gender	Marital status	Salutation
1	Male	-	Mr
2	Female	Single	Ms
3	Female	Married	Mrs
4	Female	¬K	Lady
5	¬K	-	Customer

Translation to OEL

$$Sal() = Mr \leftarrow K[db][Gender() = Male].$$
$$Sal() = Ms \leftarrow K[db][Gender() = Female] \wedge K[db][MS() = Single].$$
$$Sal() = Mrs \leftarrow K[db][Gender() = Female] \wedge K[db][MS() = Married].$$
$$Sal() = Lady \leftarrow K[db][Gender() = Female] \wedge \forall m \in MS : \neg K[db][MS() = m].$$
$$Sal() = Customer \leftarrow \forall g \in G : \neg K[db][Gender() = g].$$

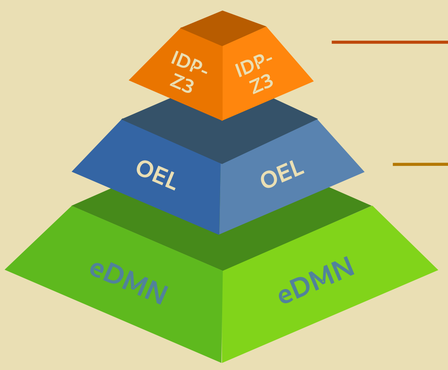
User knowledge



Decision

$Sal() = Lady$

5 Implementation [a]



IDP-Z3 [b]
Knowledge base system used as a reasoning engine

FO(OEL) [c]
Syntax extension of **FO(.)** language with hierarchical knowledge operator

eDMN [d]
Notation extension of **cDMN** with K operator

Developed tools:

- Ordered epistemic logic (OEL) solver
- OEL editor and Visual Studio code language support
- eDMN notation – Extension of cDMN with epistemic operators and translation to OEL

[a] https://gitlab.com/krr/idp-z3-oel
[b] https://www.idp-z3.be/
[c] https://dtai.cs.kuleuven.be/pages/software/idp/FO-dot
[d] https://gitlab.com/EAVISE/cdmn/cdmn-solver

6 Other results + Future

Theorem 1: *Every utility function equipped with an optimization procedure and every DMN decision function can be reduced to an epistemic decision function.*

Theorem 2: *Every epistemic decision function can be modeled as an order epistemic logic theory.*

Theorem 3: *Every epistemic decision function can be modeled as an eDMN table.*

Future work:

- Generalized OEL framework for decision modeling
- Other applications for OEL solver

References

1 Martin Peterson. 2017. An introduction to decision theory. CUP.

2 Object Modelling Group. 2023. Decision model and notation. https://www.omg.org/spec/DMN/.

3 Hanne Vlaeminck, Joost Vennekens, Maurice Bruynooghe, and Marc Denecker. 2012. Ordered epistemic logic: Semantics, complexity and applications. In Thirteenth International Conference on the Principles of KRR.

Partners

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